Algebra II: Using Quadratic Equations to Farm

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Note: This lesson has been adapted from a lesson written by Jenna Ren called *Quadratic Farmville Sweepstakes*. Jenna's lesson is available on the '*Teachers pay Teachers'* website, www.teacherspayteachers.com.

Arkansas Algebra II Content Standards:

Domain: Quantities

Clusters: 3. Reason quantitatively and use units to solve problems.

HSN.Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

Domain: The Complex Number System

Clusters: 4. Perform arithmetic operations with complex numbers.

5. Use complex numbers in polynomial identities and equations.

HSN.CN.C.7: Solve quadratic equations with real coefficients that have real or complex. solutions.

HSN.CN.C.9: Know the fundamental theory of algebra AND show that is true for quadratic polynomials.

Domain: Vector and Matrix Quantities.

Clusters: 6. Perform operations on matrices and use matrices in applications.

HSN.VM.C.6: Use matrices to represent and manipulate data (i.e.: represent payoffs or incident relationships in a network).

HSN.VM.C.8: Add, subtract and multiply matrices of appropriate dimensions.

Domain: Seeing Structure in Expressions.

Clusters: 7. Interpret the structure of expressions.

8. Write expressions in equivalent forms to solve problems.

HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context (i.e.: interpret parts of an expression using appropriate vocabulary and interpret complicated expressions by viewing one or more of their parts as a single entity).

HSA.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Factor a quadratic expression to reveal the zeros of the function it defines.
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines (standard form, vertex form and factored form).
- Use the properties of exponents to transform expressions for exponential functions.

Domain: Arithmetic with Polynomials and Rational Expressions.

Clusters: 9. Perform arithmetic operations on polynomials.

10. Understand the relationship between zeros and factors of polynomials.

11. Use polynomial identities to solve problems.

HSA.APR.A.1: Add, subtract, and multiply polynomials. Understand that polynomials, like the integers, are **closed** under addition, subtraction, and multiplication.

HSA.APR.B.2: Know and apply the Factor and Remainder Theorems: for a polynomial *p(x)* and a number *a*, the remainder on division by *x-a* is *p(a)* so *p(a)=0* if and only if *(x-a)* is a factor of *p(x).*

HSA.APR.C.4: Prove polynomial identities and use them to describe numerical relationships.

Domain: Creating Equations.

Cluster: 13: Create Equations that describe numbers or relationships.

HSA.CED.A.1: Create equations and inequalities in one variable and use them to solve problems.

Domain: Reason with Equations and Inequalities.

Clusters: 14: Understand solving equations as a process of reasoning and explain the reasoning.

15: Solve equations and inequalities in one variable.

HSA.REI.A.1: Assuming that equations have a solution, construct a solution and justify the reasoning used.

HSA.REI.B.4: Solve quadratic equations in one variable.

- Use the method of completing the square to transform any quadratic equation *x* into an equation of the form.
- Solve quadratic equations (as appropriate to the initial form of the equation) by
    1. Inspection of the graph
    2. Taking square roots
    3. Completing the square
    4. Using the quadratic formula
    5. Factoring
- Recognize complex solutions and write them as $a \pm bi$ for real numbers *a* and *b.*

Domain: Interpreting Functions.

Cluster: 19. Interpret functions that arise in applications in terms of the context.

HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval AND estimate the rate of change from a graph.

Domain: Linear, Quadratic and Exponential Models.

Cluster: 23. Construct and compare linear, quadratic, and exponential models and solve the problems.

HSF.LEA.A.2: Construct linear and exponential equations, including arithmetic and geometric sequences given a graph, a description of a relationship or two input-output pairs (including reading these from a table).

Lesson time: 90 minutes. Copy the worksheets below for the students. Have students work in groups and keep their work organized for later referral.

Elicit:

Before you begin, be sure the students have an understanding of quadratic functions (quadratic formula, regressions, solving) however, this can be covered throughout the project. Students should also know how to solve quadratic equations. If necessary, there are several websites that can help with teaching this subject such as khanacadamy.org and mathcenter.ac.uk. Have student work together as this project is better done in groups.

Engage:

Tell students they are going to figure out how to successfully grow soybeans on a farm using the quadratic formula. To make this fun, go to https://www.youtube.com/watch?v=-gwz6d9NYz0 for a song on this formula. Then have them watch the following video on growing soybeans: https://www.youtube.com/watch?v=VeBrvJNP9z0. Tell them they will be given a set number of seeds and land, and to be successful farmers, they will need to solve all the problems that go with it!

Explore:

The students will need the information below for their project.

Seed Supply

Planted Soybeans per acre	Harvested Bushels per Acre
0	0
10,000	5
40,000	20
70,000	35
110,000	55
150,000	60
190,000	50
230,000	40

Your new farm includes **five and a half (5.5)** 50-pound bags of soybean seeds (275 lbs.). You don't have time to sit there and count every bean. That's something you can look up.

Asking around the neighborhood, you are able to collect some data on how many beans per acre are planted. Using this data, you should be able to maximize your yield.

Keep in mind there are other factors involved in planting; soil depth of seed and distance from seed to seed. Assume a 2 -inch depth in the soil for each seed and 15 inches apart.

Land:

Your current block of land is 200 yards by 100 yards. But this is not necessarily the area needed for your seed. The neighbor, Farmer Fred, has agreed to let you extend your field in equal distances to the north and east by leasing some of his property, but he's charging $200/acre. Make sure you keep neighborly relations positive by requesting the correct amount of land and paying Farmer Fred correctly.

Drift Issues:

It's a windy region. When you apply nutrients to your field, you need to insure it does not get launched or drift into your neighbor's land. There are regulations in your region for drifting herbicides and/or nutrients. You have been advised to spread a potassium and phosphorous fertilizer for the soybeans. Any spray cannot be projected into the air for more than 2 seconds. Make sure your current machinery is within the regulation. It sprays the fertilizer from 3 feet off the ground with an initial downward velocity of 65 feet per second.

Elaborate and Extend:

See worksheets below. Hand these out to the students in groups to work on them for this lesson.

TIME TO MAXIMIZE YOUR PROFIT

This could make or break your future as a farmer. Your family could certainly use the extra income and there is serious satisfaction from a harvested crop. Use the information to develop a plan for your season. Use the space provide to show mathematical reasoning that defends your proposal.

Here's your collected data. Use your regression tool to find the best function model.

Planted seeds per acre	Harvested bushels per acre
0	0
10,000	5
40,000	20
70,000	35
110,000	55
150,000	60
190,000	50
230,000	40

To maximize harvest, I need to plant _____ soybean seeds per acre.

Land needs for seed supply

NOTE: Remember to convert to consistent units when doing mathematical calculations!

1 yard =_____ft 1 acre = _____ft^2 1 acre =_____rent 1 lb. soybeans = _____beans

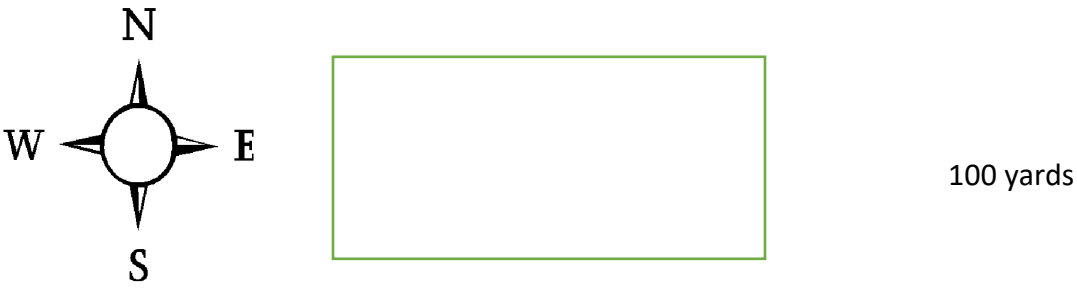
If you can figure out how many beans you have and how many acres that will cover, you can figure out how much land you need to rent from farmer Fred.

I have enough soybean seeds for _____ acres.

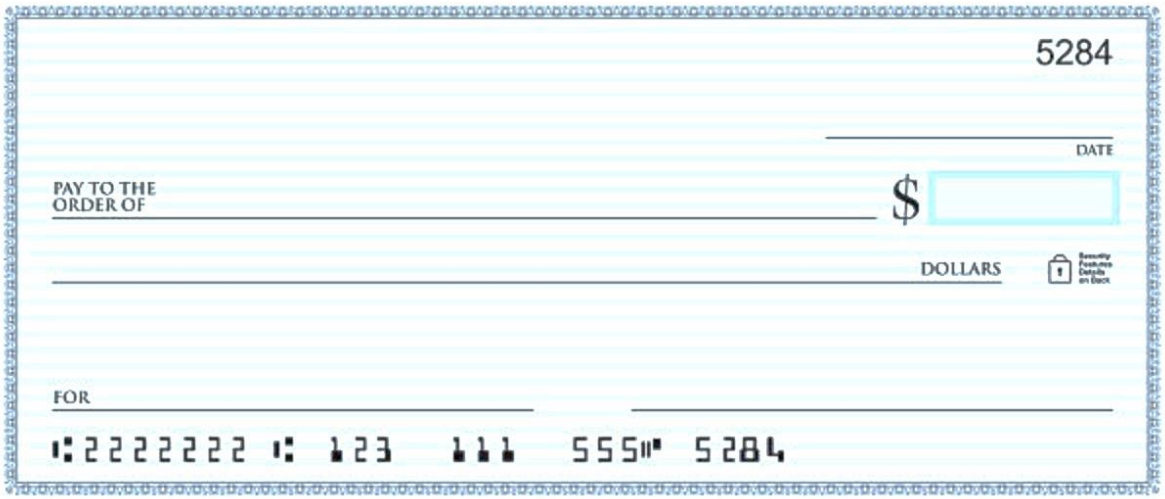
Will you need to rent land from farmer Fred? _____

Rent:

Below is the current layout of your field. Use the area formula to determine how many yards or feet you need to extend your field. You can extend it equal distances to the north and east.



Fill out the check below for farmer Fred; don't forget to write what the check is for.



Control Drift:

The height of the fertilizer sprayed from the machine as it bounces from the ground at the time (t) can be modeled by h(t) =_____

Does the upward projection of the fertilizer last more than 2 seconds? _____If it does, what can, and will you do to correct?

Final Profit:

You already know your expected bushels per acre yield from your earlier regression. By looking up the current price for soybeans, calculate your profit. What are your final thoughts? Will you proceed next year?